RAIC Centre for Architecture at Athabasca University
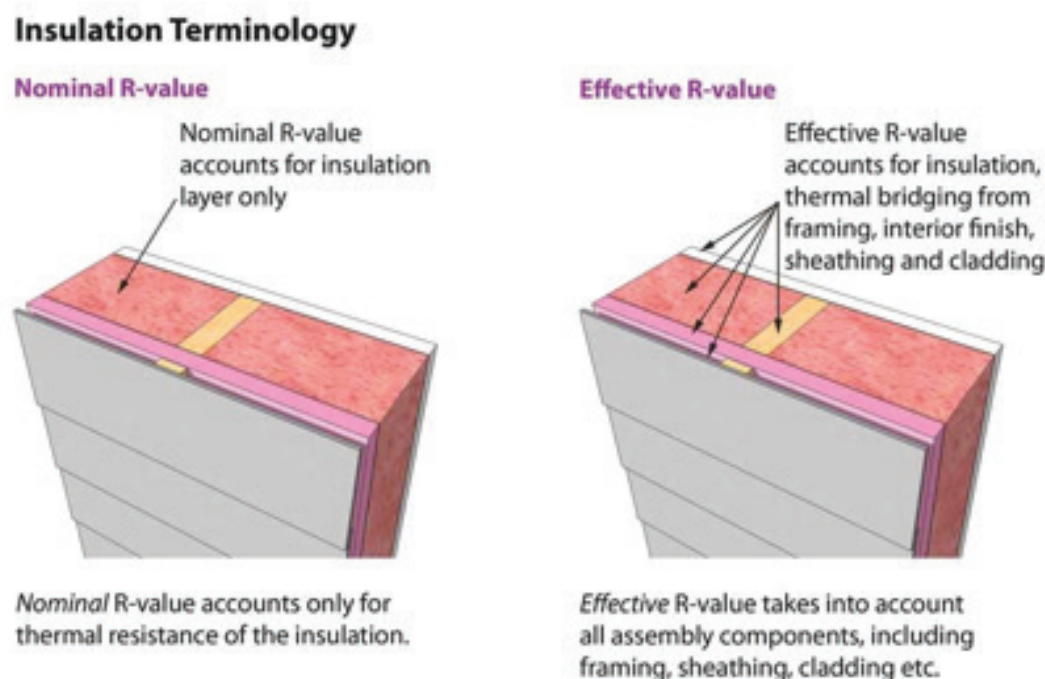
Calculating Thermal Gradients

Mapping the way that heat moves through a wall is critical to designing the envelope for any building. In fact, some building codes (such as British Columbia's) now require these calculations for your wall, ceiling and floor assemblies as part of your building permit application. Moreover, some companies now charge over $300 for this kind of work.

Another big change that occurred in the 2015 edition of the National Building Code of Canada is the move from "nominal" to "effective" values of thermal resistance. Previously, the nominal thermal resistance of a building assembly was the resistance value of its insulation layer only. Now, you must calculate the effective value - which includes the thermal resistance of all materials in the assembly.

For more on this issue you may want to visit:

https://www.constructioncanada.net/the-language-of-r-values-understanding-differences-between-nominal-and-effective/

This diagram from that website illustrates the difference:

Figure 1:
Nominal versus Effective Thermal Resistance

From:
https://www.constructioncanada.net/the-language-of-r-values-understanding-differences-between-nominal-and-effective/3/)

Self Test Question

1. Which value is likely to be higher, the nominal R value of a wall or the effective R-value? Why?
   See Answers at the end of this document.

Heat Flow

The basic idea is that heat flow through a wall or any building assembly occurs through a mixture of convection, radiation and conduction. Heat moves naturally from higher to lower areas of temperature. In general, to save energy, we want to minimize the flow of heat through a building assembly and this can be done with materials that resist heat flow (such as insulation) and by controlling air leaks.

While we typically perform this calculation through a wall section, you should be able to calculate the thermal gradient through any building assembly. In this document we'll show you how to calculate the thermal gradients through a roof, wall or any other assembly.

Some things you will need to know:

See Units of Measurement in Architecture for more information on these units.

U   The overall thermal transmittance. U is a measure of the rate at which heat is transferred through a building assembly or more specifically the amount of hear transferred through a unit area in a unit of time.

In the metric system, U is measured in W/(m2•K) where:

W   Watts (a Watt is a unit of power or energy consumption in 1 Joule of energy is consumed every second (this is the unit of time)

m2   Square meters (this is the unit of area)

K   Degrees Kelvin
Remember that a degree Kelvin is the same size as a degree Celsius – the two systems just start at different points in the temperature scale.

RSI   Is the effective thermal resistance which is the opposite or the reciprocal of U.
This gives the equation: RSI = 1/U
Because it is the reciprocal, RSI is measured in (m2•K)/W

Self Test Questions

2. The U value of an assembly is .503 W/(m2•K) what is its RSI Value?

a)   1.99 (m2•K)/W

b)   1.99 W/(m2•K)

c)   1.99 (ft2•oF.hr)/BTU

3. If resistance is the reciprocal of transmittance (RSI = 1/U) and you add up the RSI value of each material in the assembly to derive RSIT, could you just take the reciprocals of each RSI value and add them together to get U?

4. The R value of an assembly is 21.58 (ft2•oF.hr)/BTU what is its RSI value?

a)   3.88 (m2•K)/W

b)   38.8 (m2•K)/W

**c)**   0.388 (m2•K)/W

See Answers at the end of this document.

Method 1: Continuous Materials

Note: This is sometimes called the **isothermal planes** method and is used when the assembly does not have framing in the in the insulation layer.

1. Determine what units are being used. RSI or R. Because Canada is metric, RSI values will be the norm.

2. Draw or sketch a section to scale through the assembly where you want to measure the thermal gradient. Use metric units.

3. Determine if the materials continuous through the building assembly or does it consist of framing (such as wood studs) with insulation in between. If it is framed then you will need to follow **Method 2 for Framed Assemblies** (see below).

4. Make a chart of all the materials in the assembly. In the chart include columns for the Material, the Thickness, the Resistance per mm and the RSI. Work from inside to outside and don't forget the interior and exterior airfilms. Also make sure you list the materials in the order that they occur in the assembly. See Example 1 below.

5. Look at the table called **RSI Values** (provided) to copy or calculate the RSI of each material in the assembly. This table can be found at http://architecture.athabascau.ca/publications/index.php. We have kept our table as simple as possible but be forewarned every table seems to be different. See the Working with Tables Section below.

a) If the "Resistance per mm" column is blank and the "As listed" has an entry that means you can simply enter that value into the Resistance column in your chart

b) If the "Resistance per mm" column has an entry and the "As listed" column is blank then multiply the thickness of the material times the "Resistance per mm" value and enter that value into the Resistance column in your chart.

Example 1

Here's an example of a simple wall section with the thickness of each material provided:
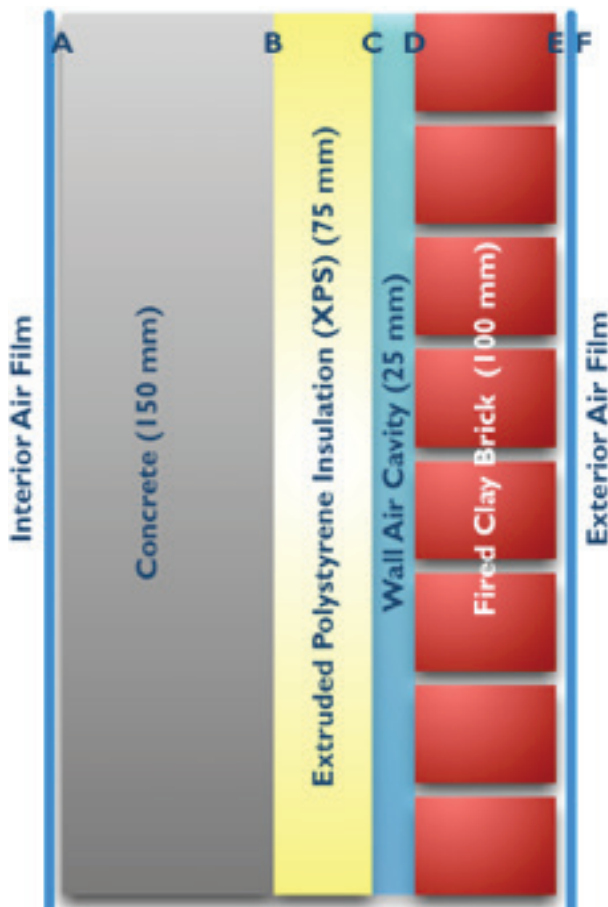
Using the table provided and the instructions in Steps 4 and 5 above, create your chart or table of materials and resistance values. Note that the Interior and Exterior Air Films are so thin that a set value has been assigned to these layers. Similarly the Air Cavity has a set value for various ranges of thickness. This is why the Resistance per mm column is blank for these entries. The values of the films and the cavity can be entered directly into your RSI column without calculations.

Figure 2: Example 1, Simple Wall Section
(The Letters A-F are referenced in Table 2)

With the concrete, XPS insulation and bricks, however, you need to multiply the thickness of each layer by its resistance per millimeter and then enter that value into the resistance column

This is shown in the table below.

			Resistance	Resistance
		Thickness	Per mm	RSI
	Units	mm	m2•K/W/mm	m2•K/W
Material				
A	Interior Air Film	n/a		0.12
B	Concrete	150	0.0004	0.06
C	Extruded Polystyrene (XPS)	75	0.035	2.63
D	Air Space	25	n/a	0.18
E	Brick	100	0.0007	0.07
F	Exterior Air Film	n/a		0.03
RSI Total				**3.09**

Table 1: Example 1, Simple Wall Section, RSI values (Shaded Values are from the RSI Values Table).

6. Once you have added all the RSI values together you should have a total RSI value for the entire assembly or RSIT. The RSIT is important so check your math.

7. Armed with this information you can then map out how the temperature changes as heat moves through the assembly. To do this you need to know the exterior and interior temperatures. Let's say the outside temperature is 0o Celsius and the inside temperature is 20o Celsius. This gives a difference of 20o Celsius (which is also the same as a difference of 20o Kelvin).

8. For each material in the assembly you can calculate the change in temperature that occurs across it:

a) First calculate what proportion of the total resistance of the assembly (RSIT) is accounted for each material in the assembly (for each material use the equation RSIi/RSIT).

b) Then multiply that proportion times the difference in temperature to give the temperature change for that material.

9. Finally, material by material subtract the temperature change from the starting temperature to arrive at the temperature in each material.

		Thickness	Per mm	RSI	Proportion		Temp. at
	Units	mm	m2•K/W/mm		Of Resistance	Temp. Change	Location in Wall
Material							
A	Interior Air Film	n/a		0.12	0.039	0.78	19.22
B	Concrete	150	0.0004	0.06	0.019	0.39	18.83
C	Extruded Polystyrene (XPS)	75	0.035	2.63	0.851	17.02	1.82
D	Air Space	25	n/a	0.18	0.058	1.17	0.65
E	Brick	100	0.0007	0.07	0.023	0.45	0.19
F	Exterior Air Film	n/a		0.03	0.010	0.19	0.00
RSI Total							**3.09**

Table 2: Temperature Changes through Wall

You can then map those temperature changes across the wall section as shown below. In this diagram I have arbitrarily decided on a vertical scale of temperature that works with the drawing.
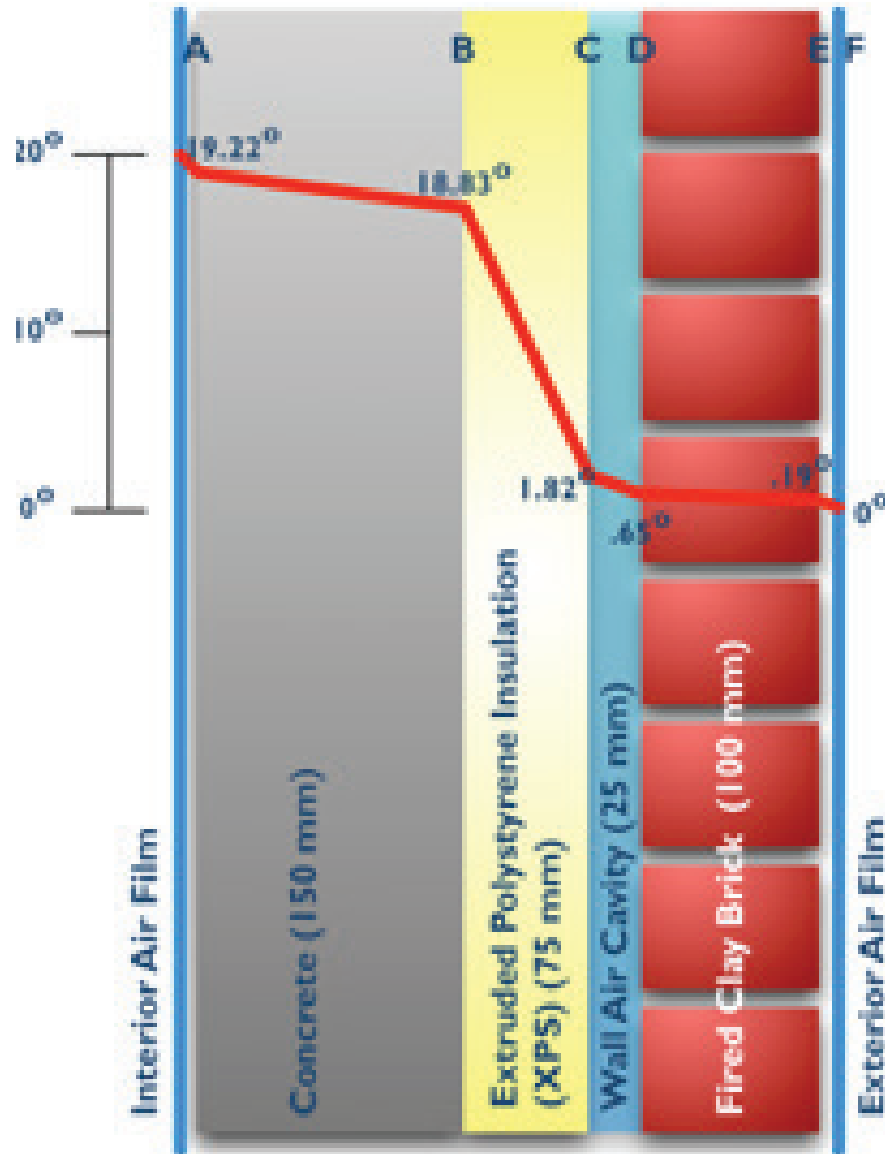

Figure 3: Simple Wall Section with Gradient

Self Test Question

5.   In Example 1, what would happen if the Extruded Polystyrene (XPS) insulation was replaced with an identical thickness of Type 1 Expanded Polystyrene (EPS) insulation. Would the RSI value go up or down? Would the U value go up or down?
See Answers at the end of this document.

Method 2: Walls with Framing Elements

As noted in the Introduction, Building Codes are now requiring designers to calculate the "Effective" thermal resistance of their assemblies. This gets complicated because framing elements such as wood studs have different thermal resistance than the insulation that may surround them.

To calculate the effective thermal resistance of a building assembly with framing elements you must use both the isothermal planes method (which you just learned), combined with the parallel path method (which you are about to learn).

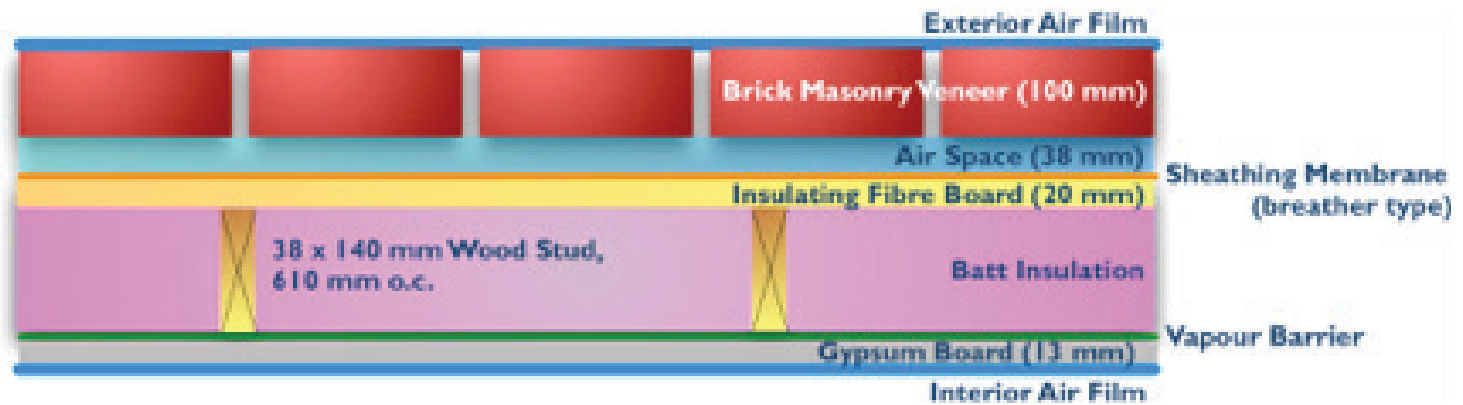Here's another wall assembly that uses wood framing:

Figure 4: Example 2, Wall with Framing Elements

Step by step method for calculating the thermal gradient through these kinds of building assemblies:

1. Again, make a chart of all the materials in the assembly. In the chart include columns for the Material, the Thickness, the Resistance per mm and the RSI. Work from inside to outside and don't forget the interior and exterior airfilms. Also make sure you list the materials in the order that they occur in the assembly.

2. Where the framing elements occur, expand the chart to include space for the parallel paths method as shown below in Example 2

3. Look at the tables provided to copy or calculate the R of each material in the assembly – including the framing elements.

   a) As before, if the *Resistance per mm* column is blank and the "As listed" has an entry that means you can simply enter that value into the Resistance column in your chart

   b) As before, If the *Resistance per mm* column has an entry and the *As listed* column is blank then multiply the thickness of the material times the *Resistance per mm* value and enter that value into the Resistance column in your chart.

4. Based on the type of framing, look up the percentages in the Wood Frame and Cavity Percentages Table which is available at: *http://architecture.athabascau.ca/publications/index.php*

		Thickness	Per mm			RSI	Proportion of	Temp.	Temp. at
Units		mm	m2•K/W/mm			m2•K/W	Resistance	K	K
Material									
Isothermal Planes									
	Interior Air Film	n/a				0.12	0.03	0.67	19.33
	Gypsum Board	13	0.0063			0.0819	0.02	0.45	18.88
	Vapour Barrier	n/a				0	0.00	0.00	18.88
Parallel Path									
	Framing Percentages			RSI's thru Stud & Cavity	%	Effective RSI			
	Wood Studs (610 mm o.c.*)	140	0.0085	1.19	20	2.80	0.78	15.55	3.33
	Batt Insulation	140		4.23	80				
Isothermal Planes									
	Insulating Fibreboard	20	0.016			0.32	0.09	1.78	1.55
	Sheathing Membrane, Breather Type	n/a				0	0.00	0.00	1.55
	Air Space	38	n/a			0.18	0.05	1.00	0.56
	Brick Masonry Veneer	100	0.0007			0.07	0.02	0.39	0.17
	Exterior Air Film	n/a				0.03	0.01	0.17	0.00
RSI Total						**3.60**			

Table 3: Example 2, Wall Section with Framing,
Green Shaded Values are from the RSI Values Table,
Orange Shaded Values are from the Wood Frame and Cavity Percentages Table.

This should give you something like Table 4 above, to calculate the effective RSI of the framed portion of the wall (the blue cell) use the following formula:

$$RSI_E = \frac{100}{((\% \text{ area of framing} \div RSI_F) + (\% \text{ area of cavity} \div RSI_C))}$$

Having determined the effective RSI of the framed portion of the assembly you then follow steps 5 through 9 from the Isothermal Planes method to determine the temperature gradient through the assembly.

Self Test Question

6. In the Wood Frame and Cavity Percentages Table, look at the percentages for walls of various types. Of the entries under "typical wood frame," "advanced wood-frame with double top plate," and "SIPS (Structural Insulated Panels)," which frame spacing will produce the greatest degree of thermal resistance if the insulation in the cavity is the same? See Answers at the end of this document.

Working with Tables

There many tables of thermal resistance figures for various materials. The problem is that each seems to use different units and approaches. When working with different kinds of tables, you should check the following:

1. Is the table working in metric (RSI's) or Imperial (R) values? See the Units of Measurement of Architecture Table if you need to convert from one system to the other.

2. If the table is metric, then you still need to check the units. Some tables list the resistance per metre and some per millimeter. If all of the dimensions in your building assembly are in mm but the resistance of the material is per metre then divide the resistance by 1000 to get the resistance per mm.

3. In some cases, the tables may only list the conductivity of the material, or k. In the metric system, k will be measured in W/m•K

Like the relationship between U and RSI, the reciprocal of k is the resistance of the material per unit of length. Again, make sure your units are consistently millimetres or metres.

This is given by the formula Resistance = Thickness/k

or

Resistance for a material of a given Thickness = Thickness/Conductivity of the material.

Self Test Questions

7. You are given the k value or conductivity of a material in W/m•K but the thickness of the elements in your assembly are in millimetres, what do you do?

8. Is k the same as K?
See Answers at the end of this document.

For Further Study

To learn more about the differences between R and U, visit "Understanding R-Value and U-Value" at http://www.norbord.com/na/blog/understanding-r-value-and-u-value

Natural Resources Canada has created a good website called Tables for Calculating Effective Thermal Resistance of Opaque Assemblies that includes some good examples in Appendix A.

http://www.nrcan.gc.ca/energy/efficiency/housing/new-homes/energy-star/14176#a52a

The Illustrated Guide for the Alberta Building Code graphically explains key terms and provides many examples. It is available for download at no cost at http://www.chbaalberta.ca/uploads/files/PDF/2016%2011%2018%20CHBA%20-%20Alberta%209.36%20Illustrated%20Guide%20%281%29.pdf

The User's Guides – National Energy Code of Canada for Buildings 2011 is another great resources with definitions, tables of materials and examples – all tied to the National Building Code. Unfortunately it is a slim document (of around 100 pages) but it costs over $100! It should, however, be a part of every architectural office. Please note as well that this document is now out of date since it refers to the 2011 Building Code which has been replaced with the 2015 version. This is, as of 2017, no illustrated guide to the 2015 Building Code has been published.

The textbook, Mechanical and Electrical Equipment for Buildings (Grondzik, Kwok, Stein and Reynolds, 11th Edition) has comprehensive lists of the k, R and RSI values for a multitude of building materials in its Appendix E. Many students will already have this book as part of their courses.

Answers to Self Test Questions

1. The nominal R value of a wall is likely to be higher because it only includes insulation which, in general, will have a higher resistance (and hence a higher R value) than the framing which typically has a lower resistance. Combining the insulation and framing values to calculate the effective R value will then result in an R value that is lower than the nominal value.

2. Obviously this is about units. 'A' is the correct answer. 'B' uses the units for thermal transmittance not resistance and 'C' uses imperial units.

3. The answer to this question has little to do with RSI's and U's but more to do with adding fractions. If you have two RSI values, a and b, then the RSIT is a+b and their U value is $1/(a+b)$. On the other hand, if you take the reciprocal of a and the reciprocal of b and add them together you get $1/a + 1/b$ which equals $(a+b)/ab$ which is not equal to $1/(a+b)$.

   You can check this with simple numbers. Let a=2 and b=3, then $1/(a+b) = 1/5$ and $(a+b)/ab = 5/6$.

4. The correct answer is 'A.' 21.58 x .18 = 3.88. The value of .18 comes from the Units of Measurement of Architecture document.

5. Looking at the chart, Type 1 EPS insulation has a per mm resistance of .026 while XPS insulation has a value of .035, therefor the RSI value will go down and the U value will go up.

6. Look for the lowest percentage of framing and the highest percentage of cavity area. In the sections specified this will be the SIPS with 14% framing and 86% cavity.

7. The simplest thing to do is convert the mm to m which is done by dividing each thickness by 1000. You could also multiply k by 1000 to change it to W/mm•K

8. No. k is a measure of conductivity while K is degrees Kelvin. While conductivity may be measured in degrees Kelvin, there is no other relationship between the two. They cannot be used interchangeably.