The Basics of Trigonometry

Trigonometry literally means the measure of triangles and it is a branch of mathematics that can be used to calculate the angles and the lengths of the sides of various triangles. It is particularly useful in analysing the force vectors of structures.

The Basics

This diagram illustrates the how and why of trigonometry.

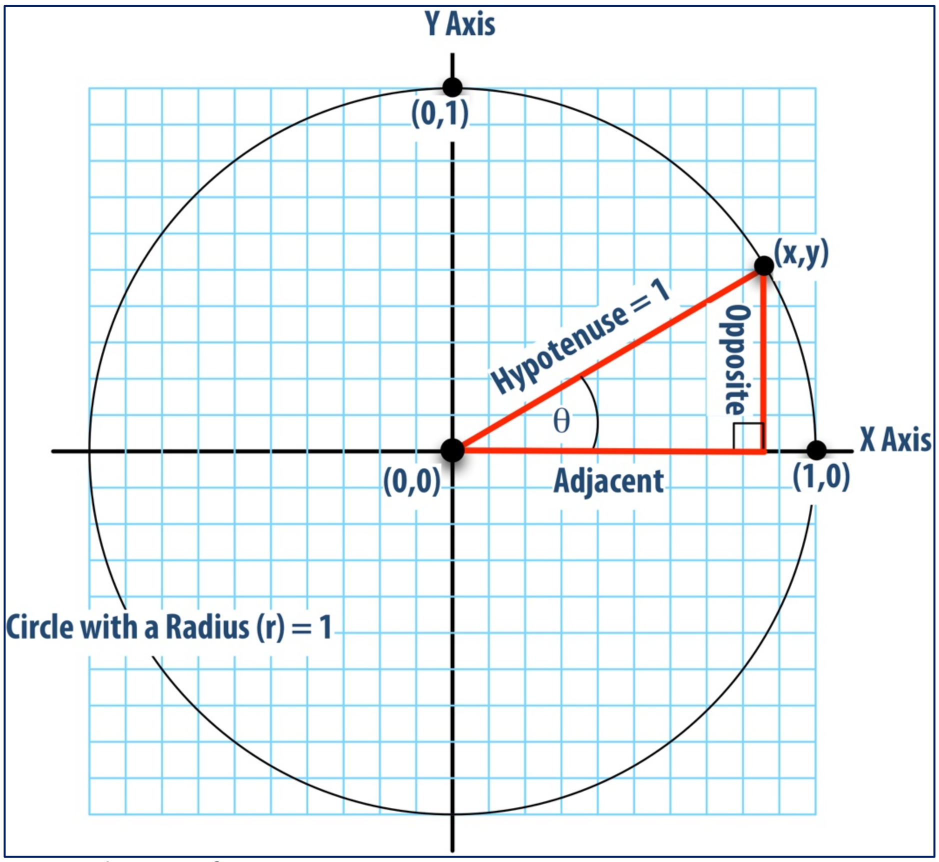

Figure 1: The Basics of Trigonometry

Imagine a circle in an x-y coordinate system with its centre at the origin (0,0) and a radius of 1. As the red lines show, any angle drawn within that circle will be part of a right-angled triangle with a hypotenuse equal to 1. The horizontal (x) and vertical (y) coordinates – (x,y) in the diagram – where the angle intersects the circle provide us with the values we need to calculate the trigonometric functions (see below) of the angle in question.

There are a number of conventions here that should be noted:

1) There are 360 degrees in a circle.
2) The angle in question is usually designated with the Greek symbol θ (pronounced 'theta'). It is just another variable or placeholder like 'x' and 'y' and it can be used in equations in the same way.
3) The magnitude of an angle is always measured in a counter clockwise direction from the x-axis.
4) The side of the right-angled triangle along the x-axis is called the Adjacent Side
5) The side of the right-angled triangle parallel to the y-axis is called the Opposite Side
6) The longest side of the right-angled triangle is called the Hypotenuse.

Note that in the diagram, the Hypotenuse is the same value as the radius of the circle or 1.

Trigonometric Functions

Using the terminology described above we can define the three main trigonometric functions: Sine (or sin), Cosine (or cos) and Tangent (or tan). These functions are just one side of the triangle divided by another or ratios of one side to another.

Sin θ = Opposite/Hypotenuse
Cos θ = Adjacent/Hypotenuse
Tan θ = Opposite/Adjacent

Equation 1: The Basic Trigonometric Functions

It is important to understand that no matter what the size of the sides of the triangle, these values are always the same for a particular angle. The tan of 45 degrees, for example, is always 1 no matter how long the sides of the triangle are.

In the diagram because the circle has a radius of 1 all values of θ in the circle will have a hypotenuse with a value of one as well. This means that in the diagram:

Sin θ = Opposite/Hypotenuse = y/1 = y
Cos θ = Adjacent/Hypotenuse = x/1 = x
Tan θ = Opposite/Adjacent = y/x

TIP: These relationships can be remembered using the mnemonic:
SOH CAH TOA for sin, cos and tan respectively.

Again looking at the diagram, you can see that the values of x and y will always be between 0 and 1 (or between 0 and -1, but we'll get to that in a minute) and this means that the value of sin and cos will also between 0 and 1. For example cos (60º) is .5.

As we noted earlier, the values of these functions are the same for any given angle so the sin and cos values of any angle will always be less than 1. Cos (60º) will always equal .5 no matter how big the triangle.

Notation: Sometimes these functions have brackets and sometimes not but sin of 45º and sin(45º) and sin 45º all mean exactly the same thing.

The values of tan, however, can vary from 0 when y = 0 to undefined when x = 0 (because you can't divide by zero). You should note that y = 0 when $\theta = 0^{\circ}$ and x = 0 when $\theta = 90^{\circ}$

Common Values

You should know some of the trigonometric values for common angles. This table below shows those values.

	0°	30°	45°	60°	90°
sin	0	.5	.71	.87	1
cos	1	.87	.71	.5	0
tan	0	.58	1	1.73	Undefined

Table 1: Common Trigonometric Values

The values that are shaded in grey have been approximated to two decimal places.

Finding the Values of Trigonometric Functions

The values of the sin, cos and tan functions used to be listed in tables in handbooks. Now a days most calculators have these functions included and even software like Excel includes the basic trigonometric functions. In a pinch you could also draw the angle on grid paper and then estimate its x and y values.

TIP: When preparing for a course in structures make sure you have a calculator with the trigonometric values.

Seeing the Patterns

You should notice in Table 1 that certain values repeat. Sin 30º for example is the same value as cos 60º. Values also repeat across the various quadrants (or quarters) of the circle. Sin 150º (in the second quadrant) is the same as sin 30º in the first quadrant. Depending on what quadrant they may be positive (+ve) or negative (-ve). This diagram shows these relationships.

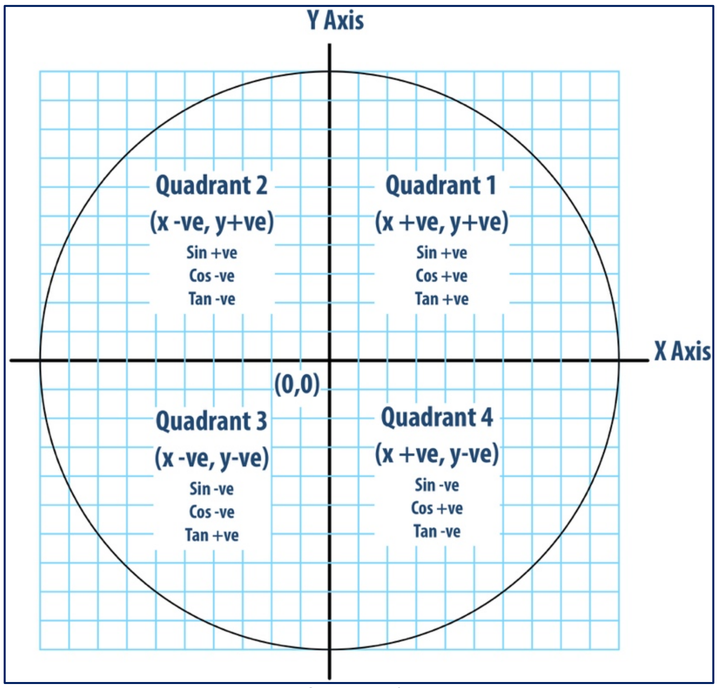

Figure 2: The Four Quadrants of a Circle (-ve = negative values; +ve = positive values)

TIP: Angles that form the same angle with the X-axis (such as 60º, 120º, 240º and 300º) will have the same magnitude for their trigonometric functions – although their sign (-ve or +ve) may be different.

Example 1: Using the Formulas

Trigonometric functions are very useful in determining the characteristics of things such as force vectors.

For example, as shown in the diagram below, a force vector of 500 kN forms a 60º angle with the horizonal. What are the horizontal and vertical components of the force vector?

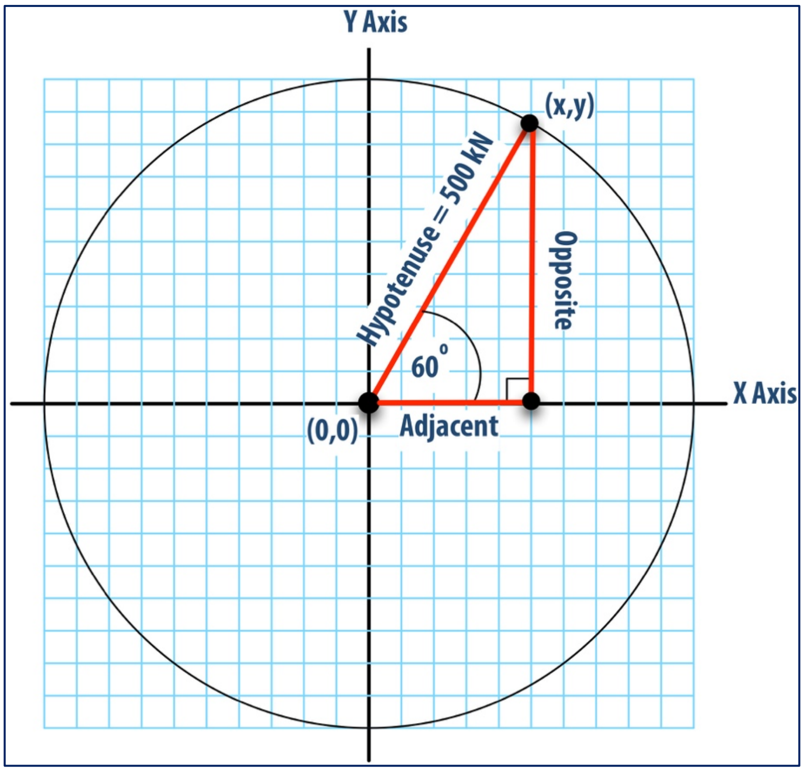

Figure 3: Example 1

For this example, you could start with either the sin or cos function. We'll use the cos function first.

As in formula listed above:

Cos θ = Adjacent/Hypotenuse

First, we substitute in the value of cos 60º (which we know from our calculator or from Table 1 is equal to .5) and the Hypotenuse (which is 500 kN as given by the question):

.5 = Adjacent/500 or
.5 = x/500

Then multiply both sides by 500 to derive:
250 = x

This means that the horizontal component of the force vector is 250 kN.

Self-Test 1: Using the Formulas

Now use the sin function to calculate the vertical component of the same force vector.

Other Useful Trigonometric Functions

The Sine Rule

The Sine Rule or the Law of Sines relates the length of the sides of any triangle (not just a right-angled one) to the sin values of its angles.

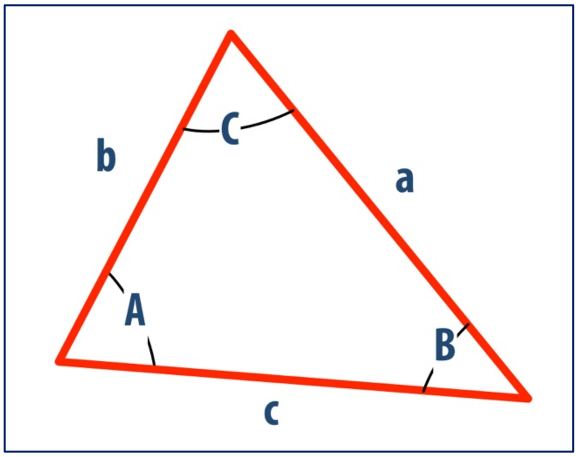

Figure 4: The Sine Rule

Using the notation of Figure 4, the formula is:

$$a/\sin(A) = b/\sin(B) = c/\sin(C)$$

Equation 2: The Sine Rule

Good examples of the Sine Rule can be found at http://www.mathsisfun.com/algebra/trig-sine-law.html

The Cosine Rule

The Cosine Rule or the Law of Cosines can be very helpful when you know the lengths of all the sides of a triangle but not the angles; or when you know the lengths of two sides of a triangle and the angle between them but you need to find the length of the third side.

Using the notation of the same diagram as the Sine Rule, it can take several forms:

$$\text{Cos}(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\text{Cos}(B) = \frac{c^2 + a^2 - b^2}{2ca}$$

$$\text{Cos}(C) = \frac{a^2 + b^2 - c^2}{2ab}$$

OR

$a^2 = b^2 + c^2 – 2bc(\text{Cos } (A))$

$b^2 = a^2 + c^2 – 2ac(\text{Cos } (B))$

$c^2 = a^2 + b^2 – 2ab(\text{Cos } (C))$

Equation 3: The Cosine Rule

Good examples of the Cosine Rule can be found at: https://www.mathsisfun.com/algebra/trig-cosine-law.html

Other Functions

Each basic function also has an inverse – but these are not used frequently. These functions are called Cosecant (Csc) , Secant (Sec) and Cotangent (Cot) and are the inverse the sine, cosine and tangent functions respectively. Their formulas are:

Csc(θ) = Hypotenuse/Opposite
Sec(θ) = Hypotenuse/Adjacent
Cot(θ) = Adjacent/Opposite

Equation 4: Other Functions

Additional Resources

The website https://www.mathsisfun.com/algebra/trigonometry.html provides a very good overview of trigonometry and it has an interactive circle that will allow you to check out the values of various angles (See the Section called "Try Sin Cos and Tan").

Self-Test Answer

The correct answer to the self-test question is:
Sin θ = Adjacent/Hypotenuse
First, we substitute in the value of sin 60º (which we know from our calculator or from Table 1 is equal to .87) and the Hypotenuse (which is 500 kN as given by the question):

.87 = Opposite/500 or
.87 = y/500
Then multiply both sides by 500 to derive:
435 = y

This means that the vertical component of the force vector is 435 kN.